Problem Chosen	2023 HiMCM Summary Sheet	Team Control Number
A		14140

Summary

The dandelion is a common plant that can now be found worldwide. However, due to its intricate relationships with the environment and other organisms, it can be considered as an invasive species, which is an organism that is alien to a region and that disrupts the local ecosystem. Invasive species can have large impacts on the environment and human societies. As invasive species exhaust local resources, not only might they decrease local biodiversity, but could also decrease crop production and thus affect the economy. Different invasive species have varying effects on the world, sometimes mild, sometimes detrimental. With this in mind, we investigated the nature of invasive species like the dandelion, modeling its spread and constructing a mathematical model in determining an impact factor, which conveys the extent of negative impact they inflict.

To begin with, we used a basic negative exponential model to predict how dandelions will spread. This model ensures that the majority of dandelion seeds would be dispersed closer to the original dandelion, however it fails to accurately simulate the trajectory of dispersal and does not take the potential of dead seeds into account. (It assumes that all the seeds dispersed would germinate and grow healthily) In order to reduce the limitations, we then incorporated the Lévy Flight Model, which is specifically used to simulate random walks and can more accurately model the paths of dispersal. After this, to gain a more realistic view, we applied a ”density-based mortality” using a logistic model, and also considered climatic factors that may affect the spread of the species. Combining these models together, we were able to simulate the dispersal of dandelions during a 12 month period.

Next, we used ranking models to determine an impact factor for invasive species. We first came up with three main determinants of this factor: ecological, economical impact, and the dispersal ability of the organism. We then researched for variables that would affect each determinant. Overall, we included 2 levels of hierarchy in determining the impact factor. Since different factors would affect the final outcome to a different extent, we combined the use of the Analytical Hierarchy Process (AHP) and Entropy Weight Method (EWM) to calculate the weights of each main determinant as well as the underlying variables in a more objective way. Then, we gave an impact score for each variable on a scale of 0 to 5, and with the use of the weights obtained previously, we were able to calculate the impact scores for each determinant and eventually, the general impact factor. We applied this model to the dandelion plant as well as two invasive species in four different climatic conditions.

From these models, we gained interesting results worth discussing. Overall, the results show that dandelions have lower impact factors in each climate compared with the other two species. In addition, a phenomenon worth noting is that in certain cases, the dispersal ability of the plant is a much more decisive factor in determining its population after a period of time. Our models suggest that invasive species are able to spread quicker than we imagine, and that they impact our world greatly in all different aspects.

Our models still need to be modified before being applied to real life scenarios, where relationships between variables are more complex. Overall, though, they are sufficiently accurate and do reflect realistic results.

Keywords: Ecology, Dandelions, Invasive species, Lévy flight model, Density-based mortality, Analytical Hierarchy Process, Entropy Weight Method

Contents

1 Introduction

Native to Eurasia, dandelions began their worldwide spread when they were first introduced to North American colonies in the 17^{th} century [1]. Since then, their numbers have multiplied rapidly. Nowadays, dandelions can be found at any part of the world. Despite having medical uses, the dandelion plant's invasive characteristics raises many concerns. Thus, in our paper, we will examine the spread of dandelions under different conditions and compute a factor showing the impact it has on the environment.

According to the reference [2], there are five distinct growing stages of a Dandelion's life: **plant**, **plant flowering**, **seed head development**, **seed development** and **dispersal of seed** through wind or other means. Specifically, when a dandelion is in its "puffball" stage, the seed is already developed and ready to be dispersed.

In the real world, many factors can affect the spread of dandelions. These include but are not limited to: wind speed, wind direction, climate, growth rate and death rate. The intricate relationships between these variables makes it difficult to replicate exactly what occurs in real life situations. However, we can make certain assumptions and create a model that can accurately estimate the spread of dandelions.

1.1 Problem Restatement and Section Overview

Firstly, we are asked to formulate mathematical models to predict the spread of dandelions across an open one-hectare $(10,000m^2)$ plot of land, over the course of 1, 2, 3, 6 and 12 months. The initial condition, as quoted from the problem, *is a single dandelion in its 'puffball' stage*, which is *adjacent to an open one-hectare plot of land.* Therefore, if we were to plot a graph where the xy plane represents the one-hectare plot of land, the initial dandelion needs to be located on either axes. In order for the model to provide an accurate estimation, it is required that we incorporate the effects of various climatic conditions. As a result, we will examine the spread of dandelions in four climates: **Tropical**, **Dry**, **Temperate**, and **Continental**.

Secondly, we are asked to create a model that computes an "impact factor" for invasive species. This model should take a holistic approach, taking into account potential benefits of the species, as well as harms they inflict on the environment. The impact factor will be a number on a scale of 0 to 5: the larger the factor is, the more negative impact the invasive plant would have. For this problem, we will first search for criteria that would influence an invasive species' impact factor. Next, ranking models would be used to give each criteria a weight based on how big of a role it plays in negatively impacting the world, and to eventually give each invasive plant an "impact factor" on a scale of 0 to 5.

1.2 Assumptions

We live in a complex world where many factors influence each other to form a final result. In order to answer the question, we must make rational, justified assumptions on certain variables.

- **Assumption 1:** The environment does not undergo significant changes.

  Justification: Random variations occur in nature, and can be minor or major. We assume that there are no major changes in environmental factors, such as natural disasters, which has a relatively low probability of occurring.

- **Assumption 2:** Basic characteristics of the dandelion plant remains the same.

  Justification: In nature, it is possible that traits of a species would change through evolution. However, this is a process that occurs over a long span of time, and a genetic variation that causes significant change in the species have a low chance of happening. For this problem, the longest time span for a prediction is 12 months, which is not nearly enough for evolution to take place. Therefore, we can reasonably assume that characteristics of a dandelion does not change.

- **Assumption 3:** Once a seed germinates, it will continue growing and will not die.

  Justification: We have computed a death rate for dandelion seeds, based on many factors including temperature, precipitation and population density. These reflect the likelihood of a dandelion surviving in its current environment after germination.

- **Assumption 4:** All the water a dandelion seed receives comes from precipitation.

  Justification: Our model takes place in a naturalistic setting, without human influence. Therefore, it is reasonable to assume that a dandelion only receives water from precipitation.

- **Assumption 5:** All of the plants studied in this paper had a germination rate (or survival rate) of 90% under ideal condition.

  Justification: The main research object of this paper dandelion has an estimated germination rate (or survival rate) of 90% [3]. Actually, for most of the plant species, a germination rate of 90% or more is desired [4]. Therefore, we can use 90% as the germination rate for plants including the invasion species discussed in Section 3.

1.3 Table of Variables

Variable	Definition
r	Distance of the dispersed seed from the parent dandelion
T	Temperature of the environment
P	Amount of precipitation per week in the environment, measured in mm/week
ρ	Population density
A	Baseline Death Rate
A_T	Base Death Rate affected by temperature
A_R	Base Death Rate affected by temperature-precipitation ratio
D	Death Rate
S	Survival Rate
M	Cost of Management and Prevention of Dandelion Invasion
E	Environmental Impact Cost
O	Social Impact Cost
B	Potential Benefit
C	Coverage of a single plant
H	Relative height of a invasive plant to native plant
N	Number of individual
M_D	Maximum density
M_S	Maximum spread distance
N_P	Impact to Native plants
N_A	Impact to Native animals
F_{Econ}	Economical Impact Factor
F_{Eco}	Ecological Impact Factor
F_D	Dispersal Impact Factor
I	Impact Factor

Table 1: Variables and Definitions

2 Modeling the Spread of Dandelions

2.1 Negative Exponential Distribution Model - A Simple Model

To start off, we created a simple negative exponential model of the spread of dandelion seeds. In this model, limited by the amount of data and for the convenience of presentation, only the dispersal of one flower head is considered. It is assumed that a single dandelion flower head can produce **200** seeds [5]. In addition, no external factors such as climate and transport of seeds by animals, are considered.

We assume that the distance of seed dispersal follows a negative exponential distribution, whose probability density function (PDF) is given by:

$$P(r) = \lambda e^{-\lambda r},$$

where r is the distance of the seed from the parent dandelion, and λ is a positive real number.

The expected value of the negative exponential distribution is $^1/_\lambda$, which is the average seed dispersal distance of the dandelion in our model. An increase in λ would result in a **decrease** in the average distance of the seeds from the dandelion. Simply put, there's a higher probability of a seed landing near the parent dandelion.

Using the negative exponential distribution, we are able to simulate the dispersal of each individual dandelion seed with the following steps:

1. Randomly select a dispersal direction, uniformly within the range $0 \leq \theta \leq 2\pi$.
2. Sample a value from the negative exponential distribution based on the probability density function. Use this as the distance travelled.
3. Obtain the landing coordinates (x, y) by multiplying the distance with $\cos(\theta)$ and $\sin(\theta)$, respectively.
4. If a seed does not land on the one-hectare plot of land, represented by the xy plane of the graph, then it is considered dead and would not be plotted on the graph.

As a result, we are able to plot the following graph, where the x-axis represents the horizontal distance from the initial dandelion[1], and the y-axis represents the vertical distance from the initial dandelion(See Figure 1).

We can tell from the graph that the dispersed seeds are more concentrated near the initial dandelion, and are spread out evenly in all directions.

Limitations:

- It only considers how the dandelion seeds might spread, but fails to consider whether the seeds would germinate or not.
- It does not consider the effects of climate.
- It does not show the spread of dandelions over a certain period of time.
- It fails to consider the occasional long-distance dispersal (LDD) of the dandelion seeds.

Despite the limitations, we can still gain a general idea of how dandelions seeds should spread from this model.

[1] The initial location is on middle of the land's left edge

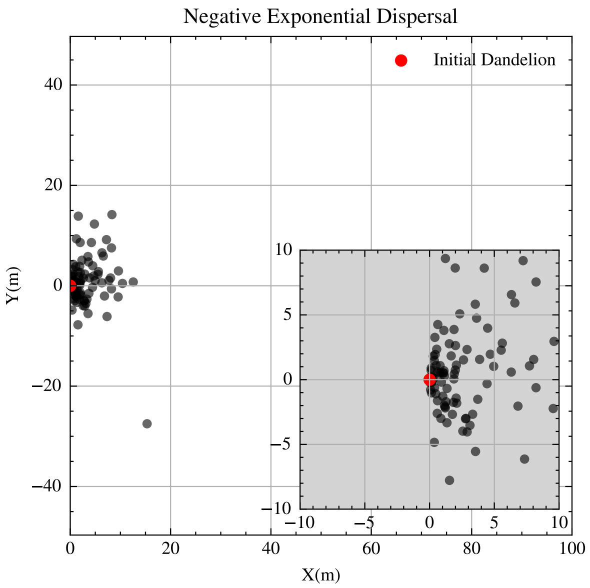


Figure 1: Simple Model of Seed Dispersal

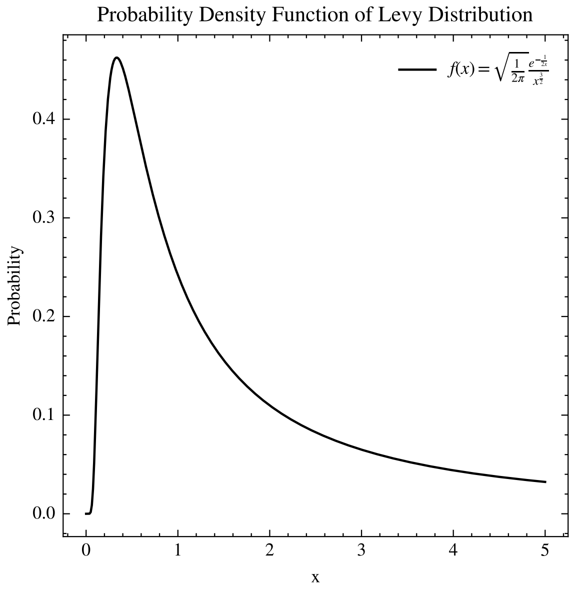


Figure 2: Lévy Distribution Curve

2.2 Lévy Flight Model

In order to obtain a better simulation of the dispersal of dandelion seeds, we applied the Lévy Flight Model. A Lévy flight is a type of random walk in which step sizes have a heavy-tailed [2] probability distribution.

In our model, each of the 200 seeds on a dandelion goes through 100 random steps before landing on the ground. The direction of each step is an angle drawn randomly from 0 to 2π; the step lengths are sampled from the Lévy distribution.

Lévy distribution is a stable distribution. Its probability density function (PDF) is:

$$f(x) = \sqrt{\frac{c}{2\pi}} \frac{e^{-\frac{c}{2(x-\mu)}}}{(x-\mu)^{\frac{3}{2}}}, \quad x \geq \mu$$

where the location parameter μ is set to 0, and the scale parameter c is set to 1.

The graph of the probability density function of the Lévy distribution, with parameters $\mu = 0$ and $c = 1$, is illustrated in the following figure (refer to Figure 2).

The complete procedure of the model is shown in the flowchart below(See Figure 5).

We pick the result of a random simulation. Figure 3 below stands for the complete path covered by the first dandelion seed that lands in the land plot, and Figure 4 shows the final landing positions of seeds.

Strength: The Lévy flight model effectively mirrors the wind-driven dispersal patterns of dandelion seeds. This model captures the tendency of the seeds to cluster primarily around the parent plant, highlighting their **central gathering** trend. Additionally, it accounts for the occasional instances where seeds are carried over long distances. This aspect of **long-distance dispersal** is crucial for understanding how dandelions spread and colonize new areas.

Weakness: Similar to the negative exponential model, no external factors such as climate, natural resource limitation are considered. Additionally, the calculation time will be unacceptable, when the

[2](A distribution that is heavy-tailed goes to zero slower than one with exponential tails [6]

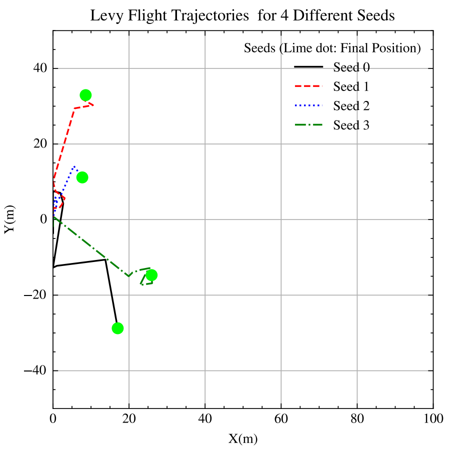


Figure 3: Path of 4 Dandelion Seeds

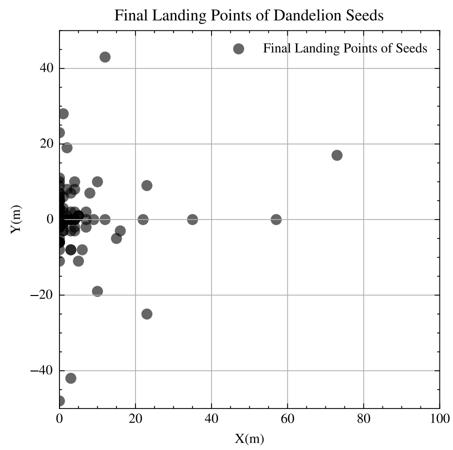


Figure 4: Dispersal of 200 Seeds

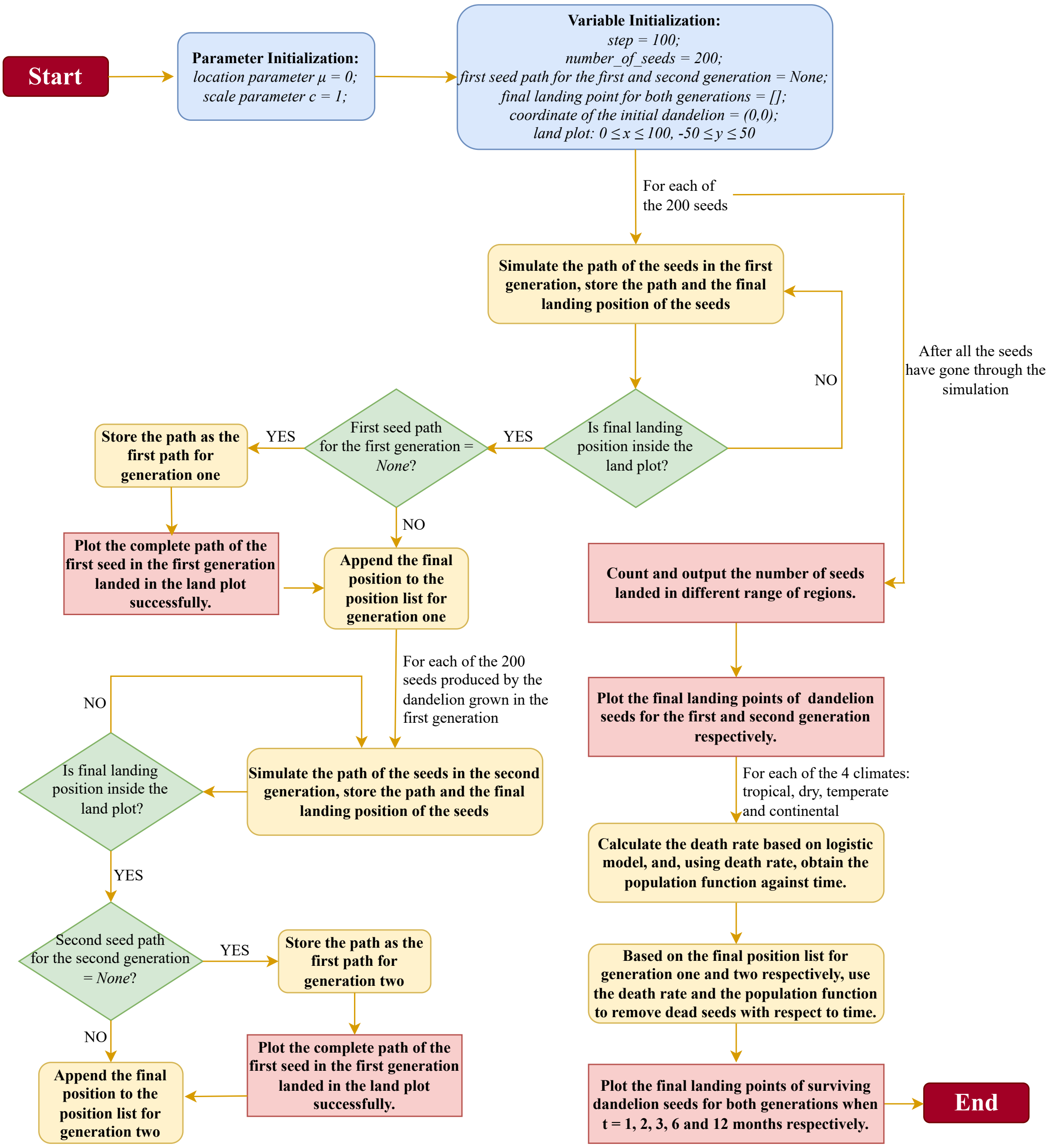


Figure 5: Lévy Flight Model Process

simulation step exceeds a certain amount[3].

2.3 Death Rate

After dandelion seeds are dispersed, it is not guaranteed that they will all survive and grow. Hence, we decide to add a death rate for each seed to more accurately estimate the spread of dandelions. In the death rate, we take into consideration exogenous factors such as climatic conditions, as well as endogenous factors such as population density. The death rate simply decides which seeds germinate and which seeds don't. We also assume that if a seed germinates, it is going to continue growing and would not die. For instance, in the context of our model, if a large number of seeds dispersed do not germinate, then the death rate will be high.

Death rate can be affected by many factors. Specifically, for our model, we decide to account for these main factors:

- **Temperature (T)**

  Dandelions have an optimal temperature range for growth. Extreme temperatures on either side of this range increase the death rate.

- **Precipitation per week (P)**

  Dandelions need water to survive. Insufficient water increases the death rate, while an optimal level maintains a standard death rate.

- **Population Density (ρ)**

  Higher population density means each plant will have less nutrients to consume, so the death rate will increase. It is measured in number of dandelions per square meter. ρ is a good indicator of resource consumption that helps us make the model more realistic.

- **Baseline Death rate (A)**

  The death rate when population density is close to zero.

What we want is a death rate function that varies within the range of $(0, 1)$ with low death rate when density is low, then death rate increases rapidly as density increase, finally reaching a saturation stage when ρ is too high.

According to the traits we need, a logistic model seems plausible.

2.3.1 Introduction of the Logistic Model

The logistic growth model is a mathematical model commonly used to describe how a population grows rapidly at first and then stabilizes asymptotically at a maximum population size over time. It is characterized by its S-shaped curve, known as the logistic curve, which represents the population size as a function of time. This model is particularly useful in ecology and biology for studying populations limited by environmental factors and carrying capacity.

To suit our needs, some modifications are required. As we have set a baseline death rate in our model, we need to make sure that when the density $\rho \to 0$, D[4] $\to A$[5], which is the ideal death rate in an ideal environment.

[3] In our simulation process, the maximum steps that our computers can calculate is 1000.
[4] Death rate
[5] Baseline death rate

Additionally, the density must have a upper limit of 1. when $\rho \to \infty$, $D \to 1$. Therefore, starting with these boundary conditions, we can derive a function.

$$D(\rho) = \frac{L}{1 + e^{-k(\rho - \rho_0)}}$$

The upper limit of D is 1, so $L = 1$

$$D(\rho) = \frac{1}{1 + e^{-k(\rho - \rho_0)}}$$

When $\rho = 0$, $D = A$.

$$A = \frac{1}{1 + e^{k\rho_0}}$$
$$e^{k\rho_0} = \frac{1 - A}{A}$$

Then we can get:

$$D = \frac{1}{1 + \frac{1-A}{A} \cdot e^{-k\rho}} \tag{1}$$

We choose $k = 0.5$ based on empirical data and for computational convenience. This choice strikes a balance between accurately reflecting real-world behavior and maintaining simplicity in the simulation.

2.3.2 Calculating Baseline Death Rate

The base death rate A is the death rate for dandelions when the population density is close to 0. For this constant, we take two factors into account : **temperature** T **and precipitation** P, since these are the two main affecting factors of a dandelion's growth, according to sources. When the climate is at the optimal temperature and precipitation rate, the dandelions would have a death rate of 10% [3]. This would also be the lowest possible value for the death rate.

As temperature and precipitation are often interrelated, it is improper to consider them separately. For instance, dandelions might thrive in moderate temperatures with sufficient rainfall, but the same temperature with too little rain could lead to drought stress, increasing their death rate. Similarly, optimal rainfall becomes less beneficial if temperatures are too low or high. Due to their interrelated traits, we decide to use a temperature-precipitation ratio for creating a more accurate and ecologically valid model.

Therefore, We decide to determine the base death rate for temperature, A_T, and temperature-precipitation ration, A_R, separately.

Firstly, we need to determine the base death rate for temperature. In general, frost can kill dandelions when temperatures reach 0 $^\circ C$ or lower. This is the temperature at which water freezes and the cold air causes the cells of the dandelion to freeze and die. So we assume at temperature below 0 $^\circ C$, the death rate of dandelion is 100% [7]. Conversely, dandelions flourish the most at about 21 $^\circ C$, which is the optimal temperature for the growth of dandelions, and would bring the lowest death rate [8]. For the temperature between 0 $^\circ C$ to 21 $^\circ C$ and the temperature above 21 $^\circ C$, we estimate that the death rate will increase quadratically with the increase of absolute difference with the optimal temperature. We then plot the death rate against temperature(See Figure 6).

The equation for A_T would be the equation for this graph, which is given by:

$$A_T = \begin{cases} 100, & \text{for } T \leq 0 \\ 0.2(T-21)^2 + 10, & \text{for } 0 < T < 42 \\ 100, & \text{for } T \geq 42 \end{cases}$$

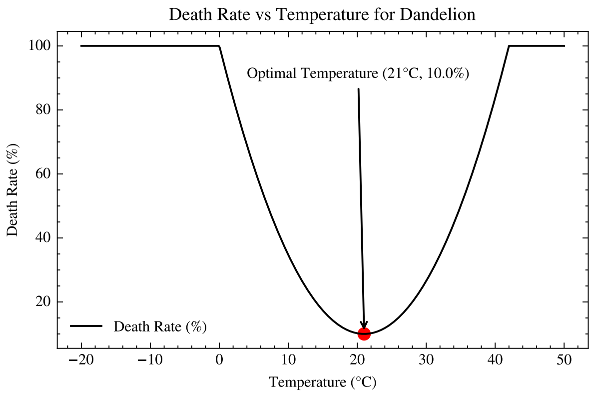


Figure 6: Death Rate vs Temperature Graph

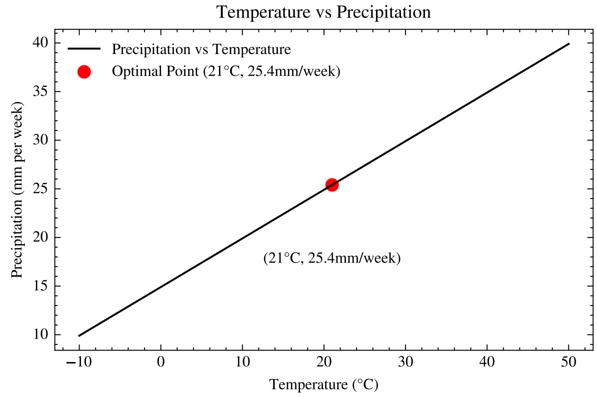


Figure 7: Precipitation VS Temperature Graph

Secondly, we need to obtain the base death rate for precipitation. For optimal growth and health, dandelions should be watered at least once a week during the summer months. Dandelions need 1 to 2 inches of water each week, which, in our model, all comes from the precipitation. During periods of dry or hot weather, they may need to be watered more often. In the winter, they typically need to be watered less frequently, only once every two weeks or so [9]. Thus, weekly precipitation requirement is related with the temperatures.

For a dandelion, the optimal condition would be:

- **Temperature:** 21 $°C$
- **Precipitation:** 25.4mm per week (1 inch per week)
- **Temperature-Precipitation Ratio:** 0.83

At this point, the death rate reaches a minimum point of 10%. For every 10 $°C$ increase in temperature, plants require an additional 5mm/week of precipitation [10]. Thus, we can deduce that there is a linear correlation between temperature and precipitation, as demonstrated in Figure 7

$$P = 0.5T + 14.9$$

The line passes through the point $(21, 25.4)$, which is the optimum condition for dandelions.

Since temperature and precipitation rate are related, it would be better to calculate the base death rate for a temperature-precipitation ratio. Here, the death rate would be 100% when the temperature-precipitation ratio is 0, and when it's 1.66 or above. We could again estimate the base death rate for ratios between 0 to 1.66 by using a quadratic growth fit (See Figure 8).

$$A_R = 1.2755 \cdot (R - 0.83)^2 + 0.1(R \neq 0)$$

where R is the temperature-precipitation ratio. **After determining the two separate components, we could find the composite base death rate by taking the average of A_T and A_R.** Hence, the composite base death rate A, which we will substitute into the death rate function, is given by:

$$A = \frac{A_T + A_R}{2}$$

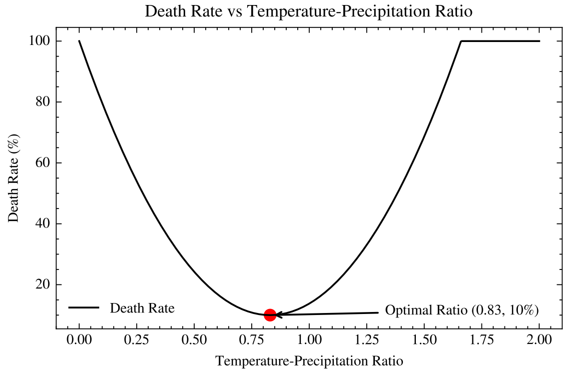


Figure 8: Death Rate VS Temperature-Precipitation Ratio

2.3.3 Adding Death Rate to the Lévy Flight Model

Currently, the Lévy flight model still does not take external factors into account. Thus, the death rate function (Explained in Section 2.2) is added to this model. The death rate would differ in different climates. We decide to consider these four climates, which are mentioned previously: **Tropical, Dry, Temperate, and Continental.**

Next, research is done on the average temperature and precipitation in each climate, as they are the key components in the death rate formula defined in the previous section. We are also able to substitute the values of temperature and precipitation above into the function for base death rate, and then the function for death rate. Specifically, Table 2 lists all the data we collected and the corresponding death rates we calculated based on these data using the models described above.

Climate	**Average Temperature** ^{o}C	**Precipitation** $mm/year$	**Death Rate** $\%$
Tropical	25 to 28 (Average 26.5)	Average 2266	19.81
Dry	18	Average 200	55.9
Temperate	-3 to 18 (Average 8)	Average 1205	22.6
Continental	10.5	Average 900	23.8

Table 2: Average Temperature & Precipitation in different Climates [11]–[18]

Hence now, having simulated the landing positions for each dandelion seed using the "Lévy Flight Model", there needs to be an additional step of deciding whether the seed would live or die. This would be done using the death rate calculated. For example, in a tropical climate, according to our evaluation, a seed would have a 19.81% chance of dying and a 80.19% of surviving.

In order to gain a more precise simulation of the seeds' dispersal, we dynamically consider the death rate, as the density varies at each location, resulting in different death rates for each location (see Figure 9). Density and mortality are considered locally. In our study, we approach the analysis of density and mortality through a grid-based method, as illustrated in Figure 10. For visual clarity in the figure, each grid represents an area of 100 m^2. However, it's important to note that in our actual calculations, we employ a much finer grid resolution, with each grid covering an area of just 0.5 m^2. This smaller grid size is crucial for ensuring the accuracy of our results, allowing for precise measurements of density and the subsequent determination of mortality rates within each grid. By adopting this method, we can accurately assess the number of seeds that die in each area, despite the randomness of their distribution.

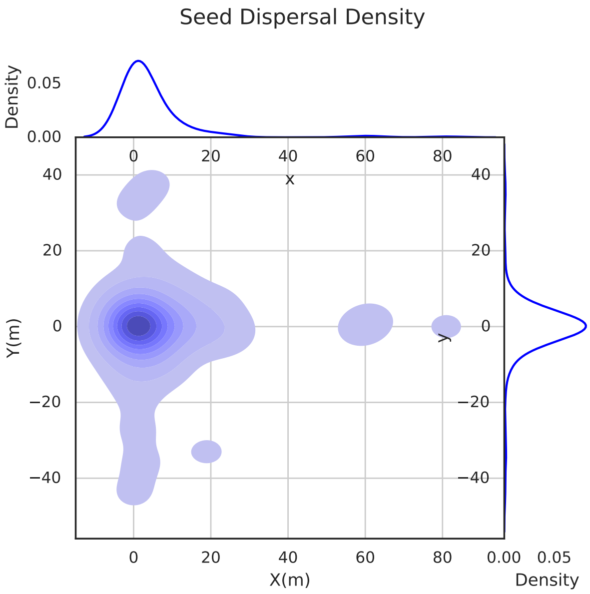


Figure 9: Dispersal Density

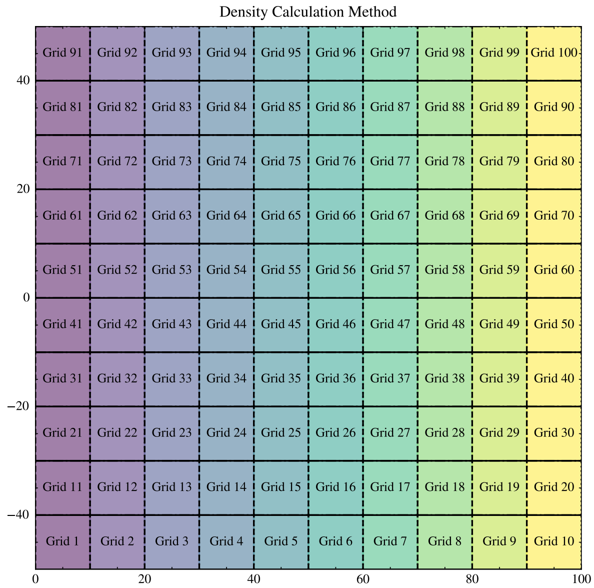


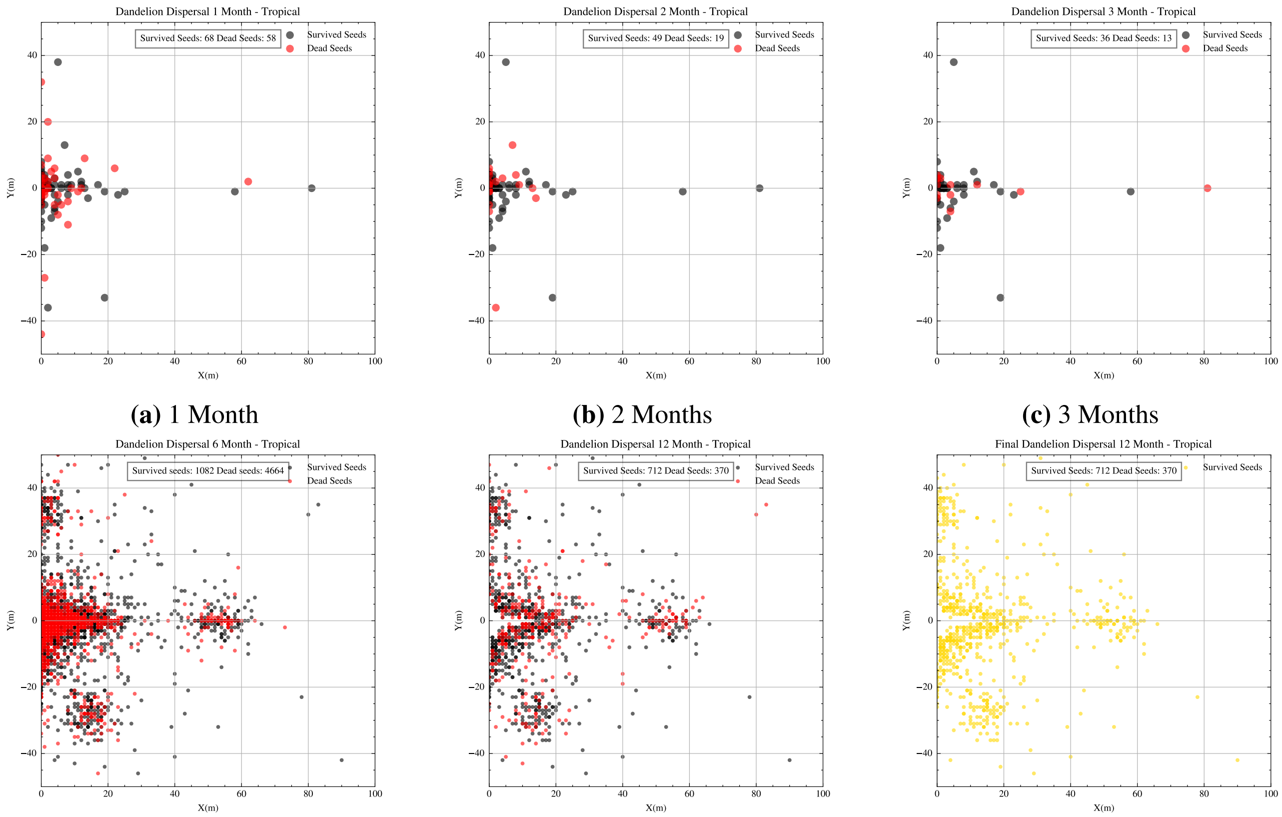
Figure 10: Density Calculation Method

2.3.4 Taking Time Into Consideration

After determining the critical models for mortality and dispersal, we can now have a brief simulation of the dandelions' dispersal throughout time. To consider time, we can use a dynamic model, which considers density dynamically, same for death rate as it is resulted from density. Let's assume that some dandelion seeds will die out everyday; therefore, the death rate on the subsequent day will be based on the previous day. The figures below show the individual numbers of deaths during each time period. The cumulative deaths overtime for each climate is shown in Table 3

Climate	Month 1	Month 2	Month 3	Month 6	Month 12
Tropical	58	77	90	4754	5124
Dry	91	107	113	1861	2007
Temperate	50	72	85	5448	5796
Continental	49	70	83	5709	6122

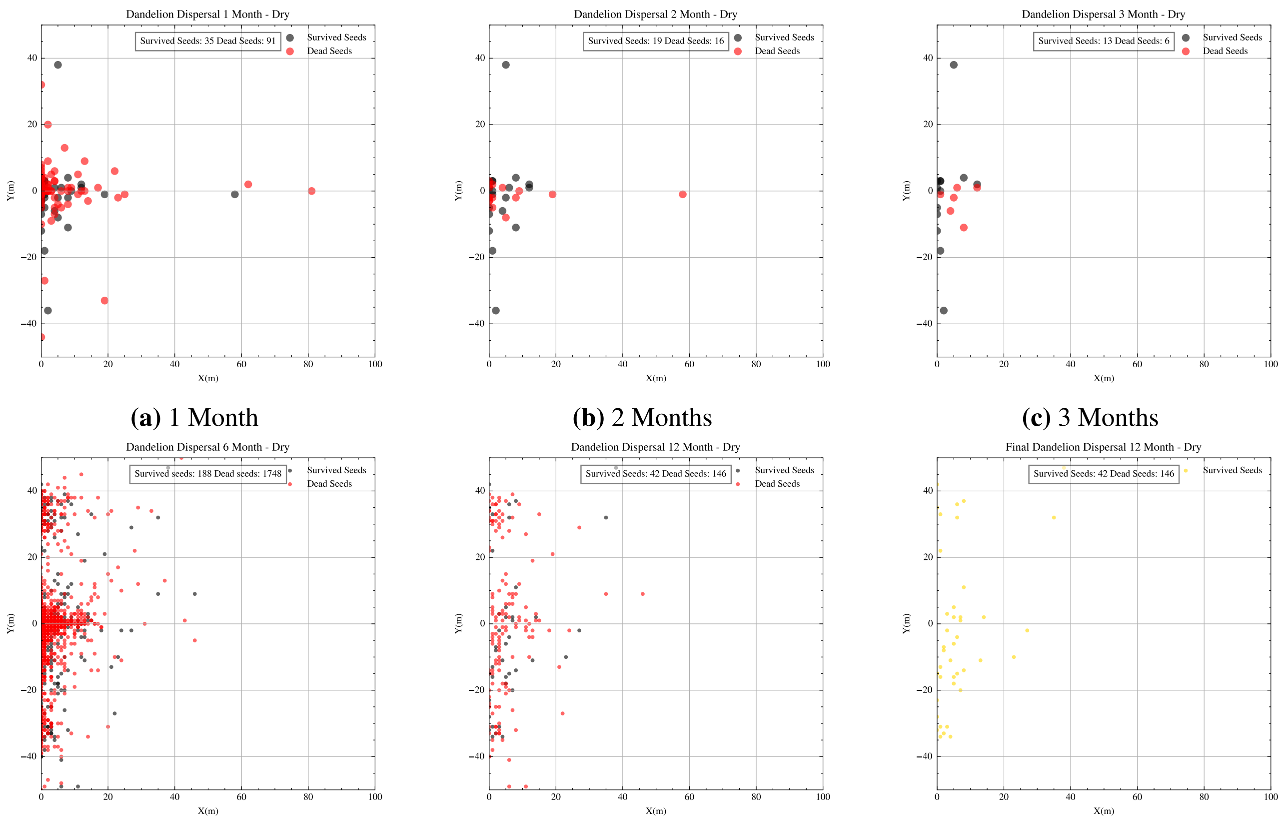
Table 3: Cumulative Death Numbers



(a) 1 Month **(b)** 2 Months **(c)** 3 Months

(d) 6 Months **(e)** 12 Months **(f)** 12 Months (Survived)

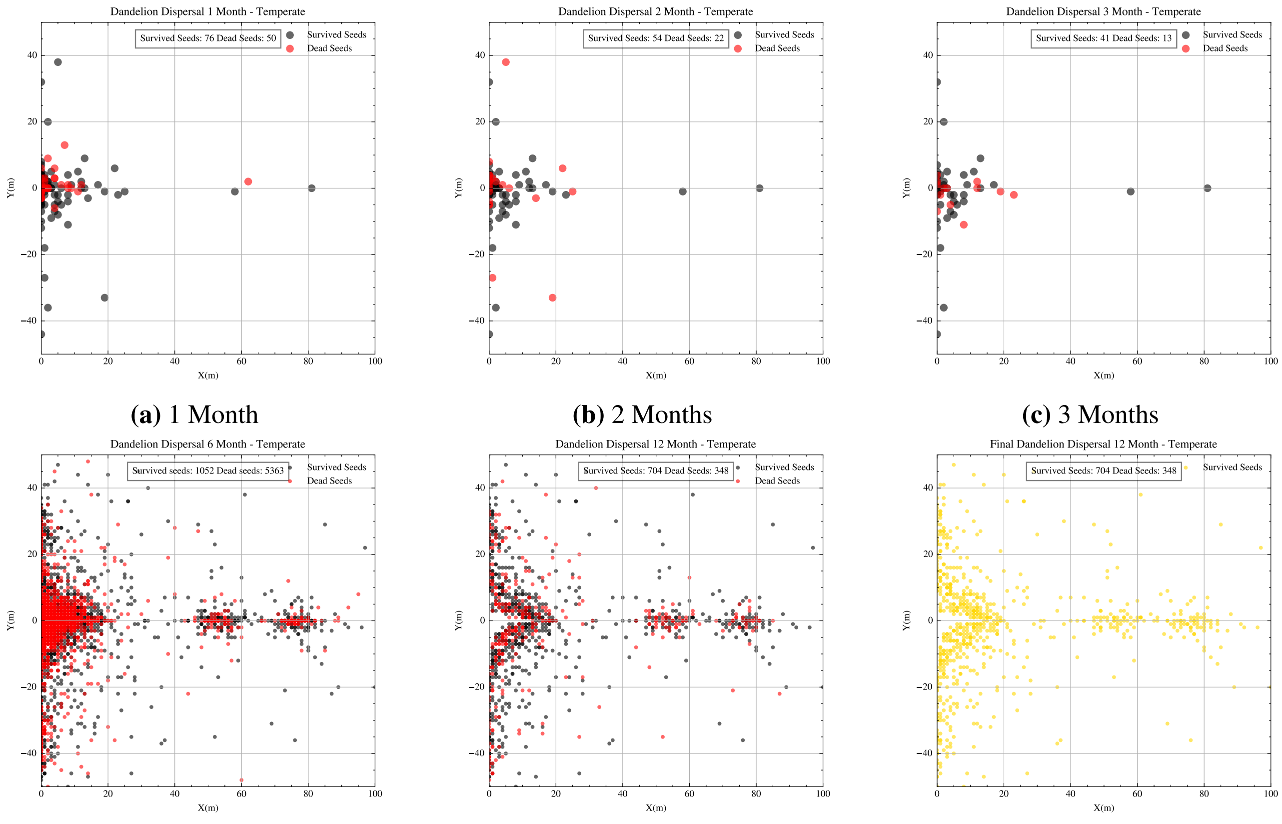
Figure 11: Dandelion Dispersal Overtime in a **Tropical Climate**



(a) 1 Month **(b)** 2 Months **(c)** 3 Months

(d) 6 Months **(e)** 12 Months **(f)** 12 Months (Survived)

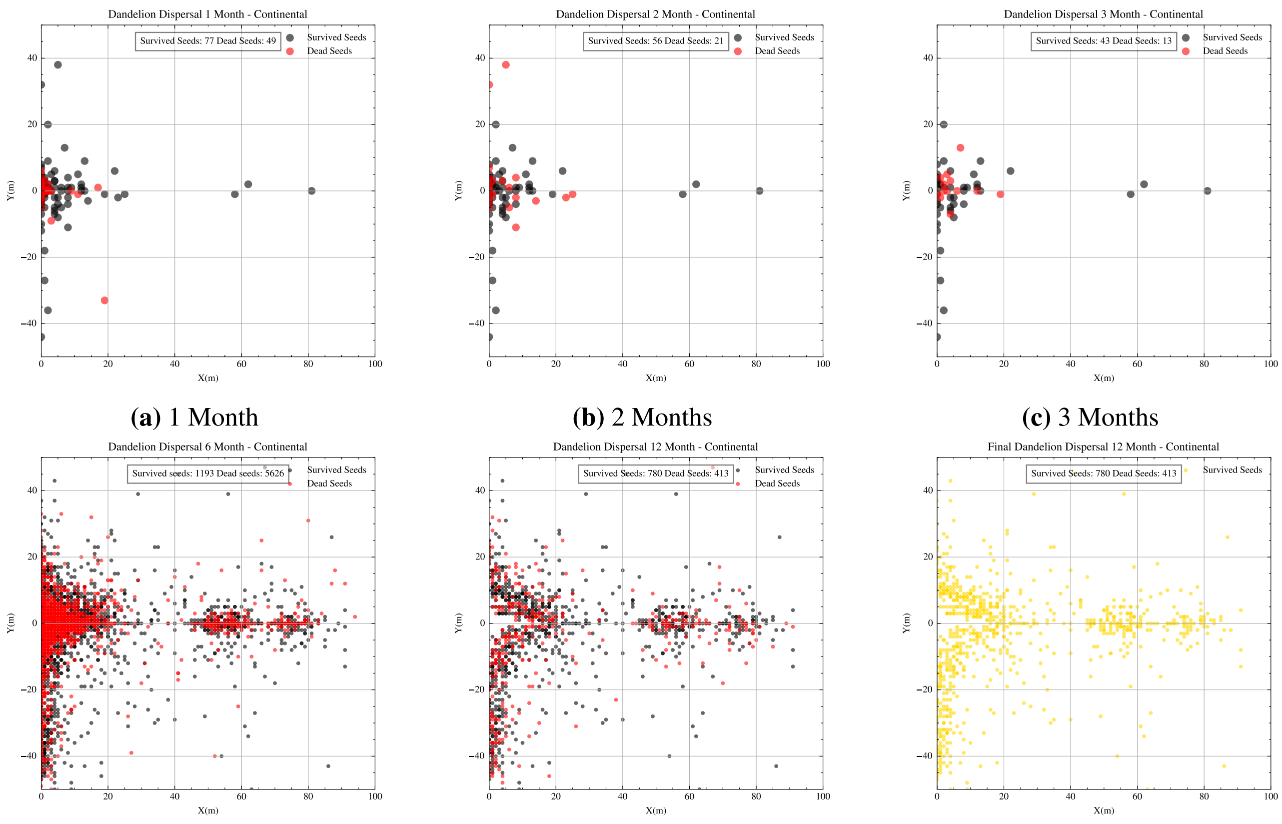
Figure 12: Dandelion Dispersal Overtime in a **Dry Climate**



(a) 1 Month **(b)** 2 Months **(c)** 3 Months

(d) 6 Months **(e)** 12 Months **(f)** 12 Months (Survived)

Figure 13: Dandelion Dispersal Overtime in a **Temperate Climate**



(a) 1 Month **(b)** 2 Months **(c)** 3 Months

(d) 6 Months **(e)** 12 Months **(f)** 12 Months (Survived)

Figure 14: Dandelion Dispersal Overtime in a **Continental Climate**

2.4 Question 1 Results Discussion

In our study, according to figs. 12a to 12f, a dry climate consistently exhibited the highest mortality rate over a 12-month period, resulting in the lowest number of surviving seeds at the end of the year (only 42 survived). Interestingly, the other three climates—tropical, continental, and temperate—showed similar patterns in seed survival(see figs. 11f, 13f and 14f), despite having different baseline mortality rates. This raised a question regarding the continental climate, which, despite having the highest baseline mortality rate (among the three), ended up with the **most** surviving seeds at 12 months.

To explain this anomaly, we hypothesized that seed density might play a significant role. In tropical climates, the lower initial mortality rate means fewer seeds die early (as can be inferred from figs. 11a to 11c). However, this leads to higher density in central grids, which in turn increases the mortality rate over time. Thus, paradoxically, a more **'suitable'** climate could result in **lower** seed survival due to density-induced mortality.

Furthermore, we noticed that seed distribution in continental and temperate climates is more **widespread** compared to the tropical climate (see figs. 11f, 13f and 14f). This dispersion could be a response to the high density and mortality in central grids. Seeds in continental and temperate climates might disperse more to seek **additional resources**, while seeds in tropical climates remain concentrated in one area, possibly due to the land's greater carrying capacity.

3 Measuring the Impact

In the second question, we are asked to determine an **Impact Factor**,I, for invasive species, taking into consideration multiple variables. We decide to complete this using ranking models. First, we would obtain weights for each variable using the Analytical Hierarchy Process (AHP) and Entropy Weight Method (EWM), representing their importance in determining the **impact factor**. Next, these weights would be used to calculate a score, conveying the extent of negative impact a particular invasive plant has. We decide to use a six level impact level grading system (See Table 4); for each variable, we rank its impact based on the system.

3.1 Problem Restatement

We need to create a mathematical model that is able to **calculate an impact factor** for invasive species, which reflects the extent of negative impact the organism has on the world. We then need to apply this model to determine the impact factors for the dandelion and two other invasive plants of our choice. In addition, we need to discuss the regions and crops each plant is invasive to.

Impact level	Impact description
0	No data available, no impacts known, not detectable or not applicable
1	Minor impacts
3	Medium impacts
5	Major large-scale impact with high damage and complete destruction
2,4	Intermediate values

Table 4: Impact Grading Table [19]

3.1.1 Species Chosen

- **Miscanthus Sinensis**: Miscanthus Sinensis, or Chinese silver grass, is a tall, perennial grass native to East Asia. While valued for ornamental use, it can become invasive outside its native range, aggressively spreading and outcompeting native plants due to its vigorous growth and high seed production.

- **Purple Loosestrife**: Purple Loosestrife is a striking perennial plant native to Europe, Asia, and northwest Africa. It is recognized for its tall, vibrant purple flower spikes and ability to thrive in wetlands. Although it's often used for ornamental purposes, Purple Loosestrife is considered invasive in North America and Australia.

3.2 Multiple Hierarchies

Since there are many variables considered when evaluating the impact of an invasive plant, we will apply multiple hierarchies. Firstly, we decide that the impact factor will be based on **three** main categories: **the ecological, economical impacts,** as well as **the dispersal abilities** of the invasive plant. Secondly, for each of the three categories, we came up with variables that would have an impact on the category it's located in. For example, the ecological impact could be sub-divided into impact on native animals and plants. We would first calculate the impact factors of each of the main categories, then, using this information, we calculate the final impact factor of an invasive plant. More specifically, below are the factors we will consider.

3.2.1 Abundance - Dispersal Impact

The concept of abundance plays a crucial role in understanding the impact of invasive species on ecosystems. Invasive species are organisms that are introduced to a new environment, where they often lack natural predators or controls. This can lead to their rapid proliferation, significantly altering the abundance dynamics within that ecosystem.

There are five criteria considered in Dispersal impact :

Coverage per plant (C) : The area covered by a single plant.

Height comparison to local plant ($H = \frac{H_{Invasiveplant}}{H_{Localplant}}$) :

An invasive species will affect the local environment greatly when its height exceeds the local plants. This means that the invasive plant will have advantages in gaining more sunlight, thus getting more energy for reproduction, and this becomes a positive feedback.

Number of individuals (N) : Number of individuals after a 12 month period.

Maximum density (M_D): Maximum density with a grid size of 0.5 m^2

Maximum spread distance (M_S) : The longest dispersal distance from the parent plant. We include this factor because, from Section 2.4, we gained that distance is more important than amount.

3.2.2 Ecological Impact

Ecological impact is an important factor that should be considered in this case. The ecological impact mainly consists of two parts: impacts on native animals and impacts on native plants.

Impacts on native animals (N_A): This is estimated by looking at how many native animal species will be either positively or negatively affected by the invasive plant.

Impacts on native plants (N_P): This is estimated by looking at how many native plant species will be either positively or negatively affected by the invasive plant. The ability of hybridization with native plants is also considered in this case as the hybridization might increase the spread of invasive plants and enhance the invasiveness at the habitat.

3.2.3 Economical Impact

Management and prevention cost (M): This is the estimated cost in preventing the spread of invasive species.

Environmental impact cost (E): This is the estimated cost in fixing the environmental damages done by the invasive species.

Social impact cost (O): This is the cost in helping citizens recover from invasion of the plants. This can include compensating farmers whose crops have been overtaken by invasive species, medical costs for citizens who suffer health issues due to the invasive plant, etc.

Reciprocal of Potential Benefit (B): When assessing the impact of an invasive plant species, it's essential to consider its potential culinary or medicinal benefits. To accurately evaluate its negative impact through Analytical Hierarchy Process (AHP), we utilize the reciprocal of these benefits. This approach implies that a species offering greater benefits will be regarded as having a lesser negative impact.

Each part is examined through the standards mentioned in Table 4. The final economical impact factor should be a value between 0 and 5.

The hierarchy in our model is demonstrated in Figure 15.

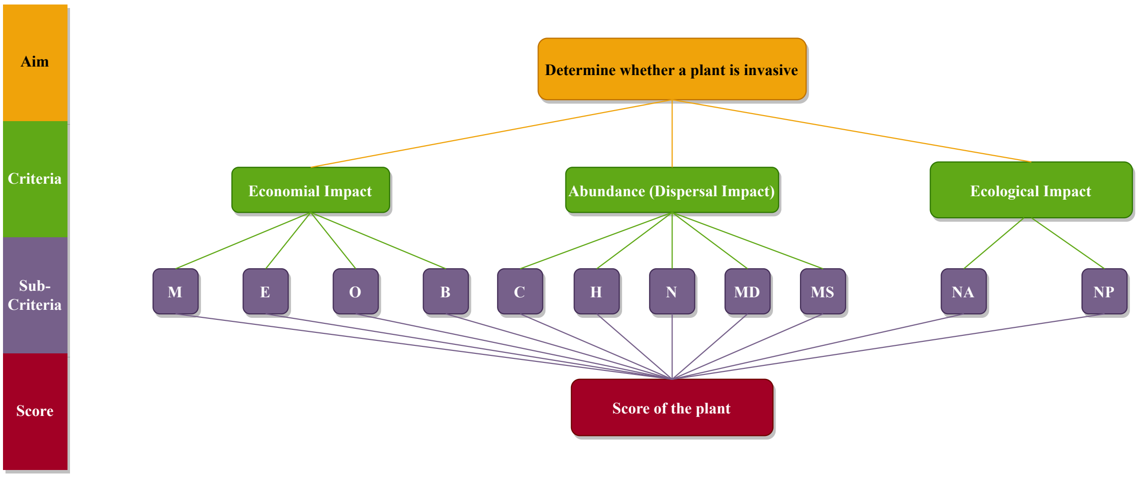


Figure 15: AHP Hierarchy

For any one of the three categories, the impact factor would be calculated by multiplying each criteria with their corresponding weights. This is denoted in the equation below, where C represents a criteria, and w represents the corresponding weights of the criterion:

$$I = \sum_{i=1}^{n} w_i C_i$$

The weights would be calculated using the AHP model.

3.3 Explanation of the AHP Model and Calculations

The AHP (Analytical Hierarchy Process) model is used to rank different factors and to calculate specific weights for each of the criteria. The weights represent their significance in affecting the final impact factor, and should add up to one. For example, a weight of 0.8 may be allocated to one of the factors, which means that it would affect the final outcome of the calculation by a large extent.

The first step in the AHP model is to determine the relative importance of the criterion to each other. For example, if we believe that factor a is a lot more important than factor b, then the relative importance of a to b may be 7. This process is mainly based on personal opinions, therefore the AHP model is a subjective model.

The relative importance is assigned based on Table 5. We first picked one criterion and decided its relative importance with the other criteria. Then, a reciprocal matrix is constructed based on these decided importance ratios. In the matrix, C_{ij} stands for the relative importance of criterion i to criterion j. To give an example, say C_{ij} equals 7, then C_{ji}, the relative importance of criterion j to criterion i, would be the reciprocal of C_{ij}, which is 1/7. Using this principle, we would be able to construct the entire reciprocal matrix, denoted below:

Relative importance	Definition	Explanation
1	Equal importance	Two activities contribute equally
3	Weak importance	Slight favor based on experience and judgment
5	Strong importance	Strong favor based on experience and judgment
7	Demonstrated importance	Strongly favored, demonstrated in practice
9	Extreme importance	Highest affirmation of favoring one activity
2, 4, 6, 8	Intermediate values	Compromise between two judgments

Table 5: AHP Scale of Relative Importance [20]

$$\begin{pmatrix} C_{11} & C_{12} & \dots & C_{1n} \\ C_{21} & C_{22} & \dots & C_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ C_{n1} & C_{n2} & \dots & C_{nn} \end{pmatrix}$$

After getting the weights, We need to check the consistency of the matrix. Let λ_{max} be the largest eigenvalue. The consistency index CI can be expressed by:

$$CI = \frac{\lambda_{max} - n}{n - 1}$$

.

Then, we compare the consistency index to random index (RI), which can be looked up in a pre-calculated table, and we obtain consistency ratio (CR):

$$CR = \frac{CI}{RI} = 0.0748 < 0.1$$

Since $CR < 0.1$, the weights are considered to be good enough. Using the weights, the impact factor for dispersal impact can be calculated.

For **dispersal impact**: $F_d = 0.28 \cdot C + 0.06 \cdot H + 0.10 \cdot N + 0.14 \cdot M_D + 0.42 \cdot M_S$

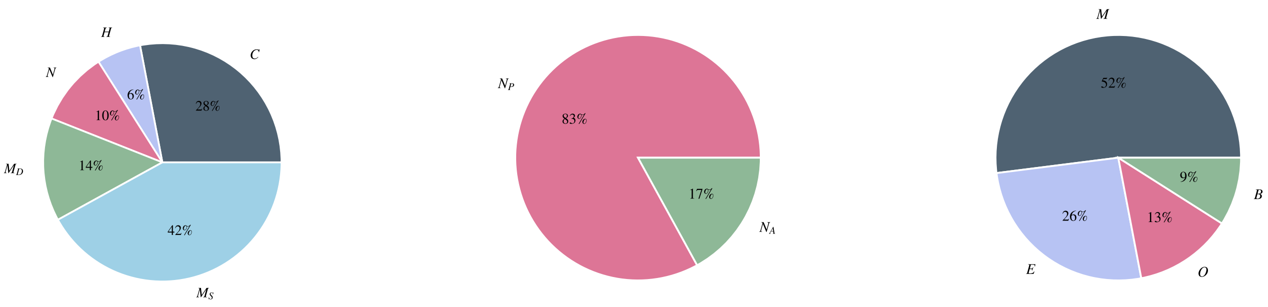
Similarly, we can get the impact factor for economial impact and ecological impact respectively.

For **economical impact**: $F_{econ} = 0.52 \cdot M + 0.26 \cdot E + 0.09 \cdot B + 0.13 \cdot O$

For **ecological impact**: $F_{eco} = 0.83 \cdot N_P + 0.17 \cdot N_A$

As for the total **impact factor** (shown in Figure 3.3):

$$I = 0.58 \cdot F_d + 0.11 \cdot F_{econ} + 0.31 \cdot F_{eco} \quad (2)$$



(a) Dispersal Impact **(b)** Ecological Impact **(c)** Economical Impact

3.4 Calculating the Impact Factor

We choose four regions with the climates we discussed before (see Table 6).

3.4.1 Dispersal Impact

We applied our model in Question 1 to simulate the dispersal of the three invasive species under four climates.

First we calculated the base line death rate A using the same process as Section 2.3.2.

In both cases, the death rate in dry climate areas is high. This is mainly because the precipitation per week is really low, which causes a high temperature precipitation ratio compared to optimal ratio. Thus, the death rate is high.

Climate Type	Characteristic Region	Local Plant	Average Height (m)
Tropical	Central America and the Caribbean	Tomato	2
Temperate	Northeastern North America	Strawberry	0.2
Dry	Southwestern United States	Prickly Pear	0.45
Continental	Central Europe	Echinacea	1

Table 6: Regions Chosen

By using the same method stated in Section 2.3.2, we could calculate the base death rate for both Miscanthus Sinensis and Purple Loosestrife. The tables below show the result:

Climate	Death Rate %
Tropical	21.4
Dry	56.2
Temperate	40.4
Continental	27.3

Table 7: Base Death Rate for Miscanthus Sinensis

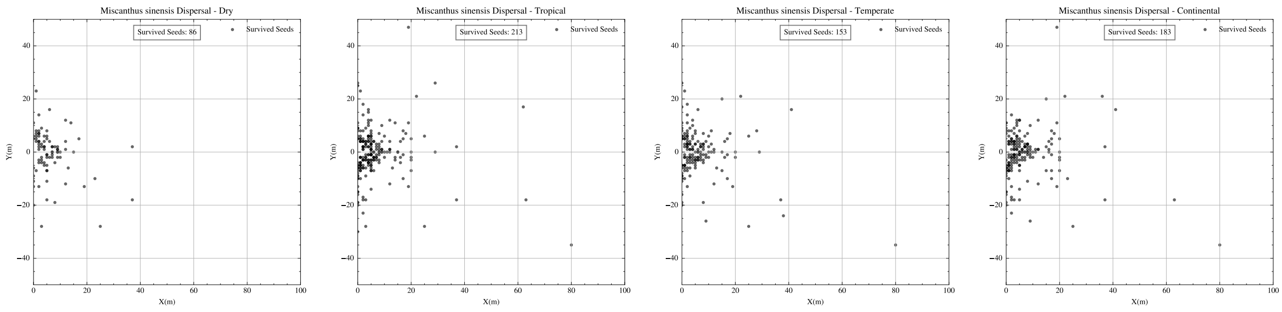
Climate	Death Rate %
Tropical	31.1
Dry	57.8
Temperate	60.7
Continental	42.4

Table 8: Base Death Rate for Purple Loosestrife

Additionally, the death rate in temperate climates is notably high. This primarily results from the average temperature in these areas being significantly lower than the optimal temperature for the two species. Moreover, the required precipitation at these lower temperatures is minimal, yet temperate regions typically experience higher annual precipitation. This imbalance in temperature and precipitation leads to an unfavorable ratio, ultimately contributing to a higher death rate.

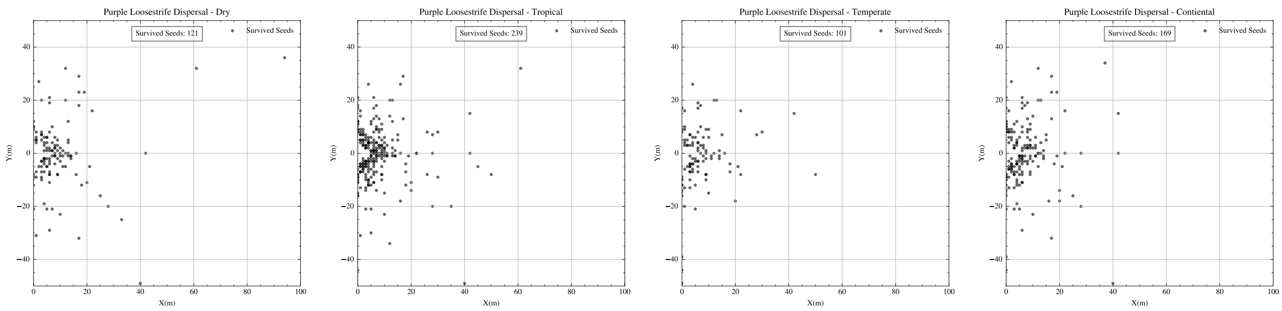
Invasive Species	Number of Seeds per Spread	Number of Spreads per Year
Dandelion	200	2
Miscanthus sinensis	2000 (Stronger reproductive capacity than dandelions)	1
Purple Loosestrife	4000 (Strongest reproductive capacity)	1

Table 9: Assumption of Reproductive Capacity of Invasive Species



(a) Dry **(b)** Tropical **(c)** Temperate **(d)** Continental

Figure 17: Dispersal of Miscanthus Sinensis



(a) Dry **(b)** Tropical **(c)** Temperate **(d)** Continental

Figure 18: Dispersal of Purple Loosestrife

3.5 Entropy Weight Method

The AHP model provides a subjective analysis. In order to offer an more objective view on the weights, we adopted Entropy Weight Method (EWM). EWM measures the degree of dispersion for different factors. To be specific, as the degree of dispersion becomes greater, more information can be obtained, and, thus, greater weight should be assigned to the corresponding index. The process of the system includes calculating the information entropy of each variables and deducing the weight of each criterion.

Since EWM cannot work based on the data of the dandelion alone, we add Purple Loosestrife and Miscanthus sinensis, two plant species that are often considered invasive, into consideration. In our model, there are 3 plants and 11 criteria. As the plant's characteristics do not remain constant in the four climates (temperate, continental, tropical and dry), there are $3 \times 4 = 12$ situations that we need to consider. The criteria, management and prevention cost (M), environmental impact cost (E), social impact cost (O), reciprocal of potential benefit (RB), coverage (C), height (H), number (N), density (MD), spread distance (MS), impact on native animals (N_A) and impact on native plants (N_p), are numbered from 1 to 11. X_{ij} stands for the value of the j^{th} criterion for the i^{th} plant (in the corresponding climate). Based on the data collected, we have the matrix X for evaluation:

$$X = \begin{pmatrix} x_{[1][1]} & x_{[1][2]} & \cdots & x_{[1][11]} \\ x_{[2][1]} & x_{[2][2]} & \cdots & x_{[2][11]} \\ \vdots & \vdots & \ddots & \vdots \\ x_{[12][1]} & x_{[12][2]} & \cdots & x_{[12][11]} \end{pmatrix}$$

In order to calculate the entropy, the matrix is required to go through the process of normalization. Let matrix Z be the matrix after normalization. Since all variables are increasing index (the greater the value of the variable, the more invasive the plant), the following formula is offered:

$$z_{ij} = \frac{x_{ij} - min(x_{1j}, x_{2j}, \ldots, x_{12j})}{max(x_{1j}, x_{2j}, \ldots, x_{12j}) - min(x_{1j}, x_{2j}, \ldots, x_{12j})}$$

The matrix Z is, thus, obtained.

Based on matrix Z, the weight p of the i^{th} option in column j is calculated:

$$p_{ij} = \frac{z_{ij}}{\sum_{i=1}^{12} z_{ij}} \quad (j = 1, 2, ..., 11)$$

Then, we use the following equation to derive the entropy of index j:

$$e_j = -\frac{1}{\ln 12} \sum_{i=1}^{12} p_{ij} \ln (p_{ij}) \quad (j = 1, 2, ..., 11)$$

The redundancy d indicates the amount of information that can be obtained from a criterion: the greater the value of d, the more information one can obtain from that criterion. It is calculated by:

$$d_j = 1 - e_j$$

Finally, we standardize d and derive the weight w_j of each criterion:

$$w_j = \frac{d_j}{\sum_{j=1}^{11} d_j} \quad (j = 1, 2, ..., 11)$$

After applying the calculation process on all the data of the three plants, we get the final weights for the criteria:

M	E	O	B	C	H	N	M_D	M_S	N_A	N_P
0.086	0.108	0.093	0.093	0.086	0.082	0.100	0.071	0.040	0.131	0.108

Table 10: EWM Final Weights

3.6 Combining AHP and EWM

Based on the final weights obtained from AHP and EWM, we need to combine these weights effectively into one final model. In other words, we need to find the appropriate proportion of AHP weights to EWM weights in the final model. Let θ_1 and θ_2 be the percentage of AHP weights and EWM weights respectively, then $\theta_1 + \theta_2 = 1$. In order to minimize the deviation between the two models, we apply Euclidean distance formula during the combination:

$$\delta = \sum_{j=1}^{11} (\theta_1 W_{AHP[j]} - \theta_2 W_{EWM[j]})^2$$

where $W_{AHP[j]}$ and $W_{EWM[j]}$ stand for the weights for the j^{th} criterion in AHP and EWM respectively.

By substituting θ_2 with $1-\theta_1$, the derivative for δ can be obtained:

$$\frac{d\delta}{d\theta_1} = 2\theta_1 \sum_{j=1}^{11} (W_{AHP[j]} + W_{EWM[j]})^2 - 2\sum_{j=1}^{11} W_{EWM[j]} \cdot (W_{AHP[j]} + W_{EWM[j]})$$

When $\frac{d\delta}{d\theta_1} = 0$, the deviation reaches its minimum. At this point, we get the corresponding optimum value of θ_1 and θ_2. $\theta 1$ has the following expression:

$$\theta_1 = \frac{\sum_{j=1}^{11} W_{EWM[j]} \cdot \left(W_{AHP[j]} + W_{EWM[j]}\right)}{\sum_{j=1}^{11} (W_{AHP[j]} + W_{EWM[j]})^2}$$

Substituting the weights into the expression, we get $\theta_1 = 0.414$, and $\theta_2 = 1 - \theta_1 = 0.586$. This means that the weights of AHP and EWM take up 41.4% and 58.6% of the final weight respectively. Then, we can get the final weight through the equation $W_{final[j]} = \theta_1 W_{AHP[j]} + \theta_2 W_{EWM[j]}$.

M	E	O	B	C	H	N	M_D	M_S	N_A	N_P
0.074	0.075	0.060	0.059	0.117	0.063	0.083	0.075	0.124	0.099	0.170

Table 11: Final Weights after the Combination of AHP and EWM

With the final weights, we can calculate the impact factor for different plant species:

$$I = W_1 \cdot M + W_2 \cdot E + W_3 \cdot O + W_4 \cdot B + W_5 \cdot C + W_6 \cdot H + W_7 \cdot N + W_8 \cdot M_D + W_9 \cdot M_S + W_{10} \cdot N_A + W_{11} \cdot N_P$$

3.7 Results

3.7.1 Impact Factor Analysis

Region	Climate	Plant	Impact factor
Northeastern North America	Temperate	Dandelion	2.11^{**}
		Purple Loosestrife	3.91^{***}
		Miscanthus sinensis	2.89^{**}
Central Europe	Continental	Dandelion	2.18^{**}
		Purple Loosestrife	3.82^{***}
		Miscanthus sinensis	2.70^{**}
Caribbean	Tropical	Dandelion	2.13^{**}
		Purple Loosestrife	3.92^{***}
		Miscanthus sinensis	2.72^{**}
Southwestern United States	Dry	Dandelion	1.26^{*}
		Purple Loosestrife	4.15^{***}
		Miscanthus sinensis	2.39^{**}
$I^* < 2, \quad 2 \leq I^{**} < 3.5, \quad 3.5 \leq I^{***}$			

Table 12: AHP Impact Factor

In all four cases, the impact factor of dandelion is the lowest, followed by Miscanthus sinensis and Purple loosestrife. This is mainly because the invasion of dandelion might bring benefits to local economy. Dandelion has been used for health purposes since ancient times and its leaves are believed to have a positive effect on the cardiovascular system due to their high potassium content [21].

For Purple Loosestrife, it has the highest impact factor among all four regions. This species possesses an high reproductive capacity, enabling it to flourish and expand over a wide range of areas. Its dense growth out-competes native species, putting native species at risk and diminishing habitat for wildlife. Economically, it obstructs waterways, hampers agricultural productivity, and hinders recreational activities, leading to substantial control and restoration costs [22].

3.7.2 Risk Assessment

Considering the above results, we set up a risk assessment criteria for invasive plant species:

Impact factor	Risk Assessment
$I < 2$	Not invasive
$2 \leq I < 3.5$	Not invasive but having moderate effect on the economy and environment
$I \geq 3.5$	Invasive

In this assessment, the dandelion is characterized as **Not invasive but has a moderate effect on the economy and environment** in Temperate, Continental, and Tropical climates. This categorization is justified because dandelions thrive in these climates, moderately outcompeting other plant species. However, in Dry climates, dandelions are classified as **Not invasive** due to their low survival rate, thus having a minimal environmental impact.

Miscanthus sinensis is also evaluated as **Not invasive but has a moderate effect on the economy and environment** across all four climatic conditions. This species tends to form substantial clumps in disturbed areas, moderately displacing native flora. Nonetheless, its use as an ornamental plant somewhat mitigates its impact.

Purple Loosestrife is deemed **Invasive** in all climate conditions, attributed to its robust dispersal capabilities and significant detrimental effects on local ecosystems and economies, according to Section 3.7.1.

3.8 Strengths and Weaknesses

Strengths: In our study, we have integrated the Analytical Hierarchy Process (AHP) with the Entropy Weight Method (EWM), resulting in a more objective approach to assigning weights in our analysis. To ensure a comprehensive understanding, we selected four distinct regions, each characterized by a unique climate. This diverse selection provides a thorough view of our research scope. Furthermore, this approach is closely aligned with the initial question posed in our paper, demonstrating a consistent thread throughout our research and reinforcing the reliability of our findings.

Weaknesses: Our analysis encounters certain limitations. Firstly, there is a lack of sufficient information to accurately establish the baseline death rate, which is crucial for our study. Secondly, even within the same climate region, temperature and precipitation can vary significantly from one location to another. By focusing only on four main climate types, our approach might not capture these nuances, leading to potential imprecision. Additionally, we acknowledge that our model doesn't incorporate enough factors to fully represent the complexities of real-world scenarios, which could affect the accuracy of our assessment of the total impact factor.

4 Conclusion

In Section 2, we provided our solution to problem 1. First, we created a simple model, simulating the spread of dandelions using the negative exponential distribution. However, this does not accurately model the path a dandelion seed takes during its dispersal, nor does it take into account any external factors. Thus, we then utilized the lévy flight model, which uses the lévy distribution curve instead. This model more accurately presents the path a dandelion seed takes during its dispersal. This model is still insufficient, though, as it still does not take into account external factors, nor does it model the spread of dandelions overtime. Therefore, we constructed a death rate that is influenced by factors such as temperature, precipitation or population density using the logistic growth model. With this death rate, we were then finally able to properly estimate the spread of dandelions overtime, using a dynamic model where the death rates in each condition are updated daily. Using this model, we were able to model the spread of dandelions over 1, 2, 3, 6, and 12 months in tropical, dry temperate and continental climates.

In section 3, we provided our solution to problem 2. We first split the determination of the impact factor into different components due to there being many variables that should be considered. Next, since different variables have various extents in determining the final impact factor, we decided to assign weights to each of them using the Analytical Hierarchy Process (AHP) and the Entropy Weight Method (EWM) combined to obtain an objective result. It was decided that there would be three main determinants of an invasive plant's impact factor: its economical, ecological impacts, and its dispersal ability. These were then further split into smaller criteria that would affect the three determinants. We used the weights obtained using the AHP and EWM models to calculate individual impact scores for each of the determinants. In the end, we combined these scores, yielding a final impact factor ranging from 0 to 5. On top of this, we determined that if a plant has an impact factor that is greater or equal to 3.5, it is considered invasive. Our results show that dandelions and Miscanthus sinensis are not considered invasive but have moderate effects on the economy and environment, whereas the Purple Loosestrife is considered invasive in all four climatic conditions.

Even though our models are rather basic compared to those constructed in real world scenarios, there are still interesting phenomena worth noting. One major characteristic we observed is that even though the death rate in a tropical climate is lower compared to a continental climate, after 12 months there were more surviving seeds in the continental climate. This lead us to the conclusion that perhaps when the death rate of two invasive plants are not significantly different, the distance of dispersal is a much more decisive factor in determining their eventual population.

To conclude, invasive species can have significant impacts on the environment and our societies. During our modeling process, we have witnessed the destructive capabilities of certain invasive plants. Therefore, the issue of invasive organisms should not, and cannot be underestimated.

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REPORT ON USE OF AI

GPT3.5 Jan 2022 version

Query1: do you know levy flight model ?

Output1:Yes, I'm familiar with the Levy flight model. The Levy flight is a type of random walk or stochastic process that describes a series of steps or jumps in which the step sizes follow a probability distribution with heavy tails, meaning that there is a small probability of very large steps. In other words, Levy flights are characterized by occasional long jumps or leaps, which can lead to a more efficient exploration of space compared to simple random walks.

The Levy flight model is often used to describe various natural phenomena, including the movement patterns of animals, the foraging behavior of organisms, and the search strategies in optimization algorithms, among other applications. It has been used in fields such as ecology, physics, and computer science to model and understand processes where intermittent long-range movements or events are significant.

Query2:Can you use levy flight to help me simulate the dispersal of 1 seed , starting from (0,0). stimulating 100 steps for the seed, and draw out the final results with matplotlib.

Output2:Certainly, I can help you simulate the dispersal of a seed using a Levy flight model and then visualize the results using Matplotlib in Python. Here's a Python code example to achieve this:

Listing 1: Seed Dispersal Simulation (Python)

```
import numpy as np
import matplotlib.pyplot as plt

# Parameters for Levy flight
n_steps = 100  # Number of steps
alpha = 1.5    # Exponent for Levy distribution (adjust as needed)

# Initialize the position at (0, 0)
x, y = [0], [0]

# Simulate the Levy flight
for _ in range(n_steps):
    # Generate random step sizes from a Levy distribution
    step_length = np.random.pareto(alpha)  # Levy distribution with exponent alpha
    step_angle = np.random.uniform(0, 2 * np.pi)  # Random angle between 0 and 2*pi

    # Calculate the new position based on the step size and angle
    new_x = x[-1] + step_length * np.cos(step_angle)
    new_y = y[-1] + step_length * np.sin(step_angle)

    x.append(new_x)
    y.append(new_y)

# Plot the results
plt.figure(figsize=(8, 8))
plt.plot(x, y, marker='o', linestyle='-', markersize=5)
plt.title(f'Seed Dispersal Simulation (Levy Flight, {n_steps} Steps)')
plt.xlabel('X-axis')
plt.ylabel('Y-axis')
plt.grid(True)
plt.show()
```

In this code: We specify the number of steps n$_steps and the exponent alpha for the Levy distribution. You can adju$

We initialize the position of the seed at (0, 0).

We simulate the Levy flight by generating random step sizes from a Pareto distribution (which is a common choice for Levy flights) with the specified exponent alpha. We also generate a random angle between 0 and 2π for each step.

We calculate the new position of the seed based on the step size and angle and append it to the x and y lists.

Finally, we use Matplotlib to plot the trajectory of the seed dispersal.

You can run this code to simulate and visualize the dispersal of a seed using a Levy flight model with 100 steps. Adjust the parameters as needed for different simulations.